Statistics

111. The following frequency distribution shows the daily savings of 64 children in a locality:

Daily savings (in Rs)	1-3	3-5	5-7	7-9	9-11	11-13	13-15
Number of children	7	6	x	13	y	5	4

If mean savings is Rs 8 then find the missing frequencies x and y.

Daily savings (in Rs.)	1-3	3-5	5-7	7-9	9-11	11-13	13-15	Total
Frequency (f)	7	6	x	13	y	5	4	35+x+y=64
x_i	2	4	6	8	10	12	14	
$f_i x_i$	14	24	6x	104	10y	60	56	258+6x+10y

$$+ y = 64 - 35 = 29 \qquad ....(1)$$

Also, $\frac{258+65x+10y}{64} = 8.$

$35 + x + y = 64$

$\Rightarrow$ $\Rightarrow 258 + 6x + 10y = 512$
x

$\Rightarrow$ $6x + 10y = 512 - 258 = 254$

$\Rightarrow$ $3x + 5y = 127$ .... (2)
So, we have:

From (1) $5x + 5y = 145$ .... (3)

Subtracting (2) from (3), we get

$$2x = 18 \qquad \Rightarrow x = \frac{18}{2} = 9$$

So from (1), we get $9 + y = 29 \Rightarrow y = 20$

---

112. In apenditure of 40 families on fruits and vegetables (in rupees) is given in the following frequency distribution:

Expenditure (in Rs.)	500-700	700-900	900-1100	1100-1300	1300-1500
Number of families	6	8	10	9	7

Find the mean weekly expenditure.

Expenditure (in Rs)	500-700	700-900	900-1100	1100-1300	1300-1500	total
frequency	6	8	10	9	7	40
x_i	600	800	1000	1200	1400	
$u_i = \dfrac{x_i - a}{h}$	-2	-1	0	1	2	
$f_i u_i$	-12	-8	0	9	14	$\sum f_i u_i = 3$

Here, $a = 1000, h = 200$

So, mean $= a + \dfrac{\sum f_i u_i \times h}{\sum f_i} = 1000 + \dfrac{3}{40} \times 200$

$= 1000 + 15 = 1015$

Thus, mean expenditure is Rs.1015

113.On the sports day of a school, 300 students participated. Their ages are given in the following distribution:

Age (in years)	5-7	7-9-	9-11	11-13	13-15	15-17	17-19
Number of students	67	33	41	95	36	13	15

Find the mean and mode of the data.

Age (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19	Total
Frequency	67	33	41	95	36	13	15	300
x_i	6	8	10	12	14	16	18	
$u_i = \dfrac{x_i - a}{h}$	-3	-2	-1	0	1	2	3	
$f_i u_i$	-201	-66	-41	0	36	26	26	$\sum f_i u_i$

$$= -201$$

Here, a= 12 $and\ h = 2$.

So, mean $= 12 - \dfrac{201 \times 2}{300} = 12 - \dfrac{402}{300} = 12 - 1.34 = -10.66$

For the mode, we have modal class$= 11 - 13$

So, mode $= 11 + \dfrac{(95-41) \times 2}{2 \times 95 - 41 - 36}$.

$= 11 + \dfrac{54 \times 2}{190 - 77} = 11 + \dfrac{108}{113}$

$= 11 + 0.95 = 11.95$

114. Calculate the mode of the following distribution table:

Marks	No. of students
25 or above 25	52
35 or above 35	47
45 or above 45	37
55 or above 55	17
65 or above 65	8
75 or above 75	2
85 or above 85	0

2015/2016 (4 Marks)

We first convert the given cumulative distribution to an ordinary cumulative distribution as shown below:

Mark (Class)	Frequency
25-35	5
35-45	10
45-55	20
55-65	9
65-75	6
75-85	2

Here, modal class is 45-55.

$$So, \qquad f1 = 20, \qquad f0 = 10 \ and \ f2 = 9$$

Hence, $\quad$ mode $= 45 + \frac{(20-10)}{2 \times 20 - 10 - 9} \times 10.$

$$= 45 + \frac{10 \times 10}{40 - 19} = 45 + \frac{100}{21}$$

$$= 45 + 4.7 = 49.7$$

---

115. Find mode using an empirical relation, when it is given that mean and median are 10.5 and 9.6 respectively.

2015/2016 (1 mark)

The empirical relation is

$\qquad$ 3 Median = Mode + 2 Mean

So, we have: $\quad 3 \times 9.6 = Mode + 2 \ \times 10.5$

$\Rightarrow \qquad\qquad 28.8 = $ Mode + 21

$\Rightarrow \qquad\qquad$ Mode $= 28.8 - 21 = 7.8.$

116. Given below is a frequency distribution table showing daily income of 100 workers of a factory:

Daily income of workers (in Rs)	200-300	300-400	400-500	500-600	600-700
Number of workers	12	18	35	20	15

Convert this table to a cumulative frequency table of 'more than type.

2014/2015/2016   (2 Marks)

Required cumulative frequency distribution table of more than type is as shown below:

Daily income of workers (in Rs)	More than or equal to 200	More than or equal to 300	More than or equal to 400	More than or equal to 500	More than or equal to 600	More than or equal to 700
Number of workers	100	88	70	35	15	0

117. Some students of class X donated for the welfare of old age persons. Their contributions are shown in following distribution:

Amount (in ₹)	0-20	20-40	40-60	60-80	80-100
Number of students	5	8	12	11	4

Find median and mode of their contributions.

2014/2015/2016   (3 Marks)

We have the following table:

Amount (in ₹)	0-20	20-40	40-60	60-80	80-100
frequency	5	8	12	11	4
Cumulative frequency	5	13	25	36	40

Here, $N = 40$. So $\frac{N}{2} = 20$

Therefore, median class is 40-60.

So, median $= 40 + \frac{(20-13) \times 20}{12} = 40 + \frac{7 \times 20}{12}$

$= 40 + \frac{35}{3}$

$$= 40 + 11.66$$

Median = ₹ 51.66

For mode, we have: Modal class 40-60.

So, mode $= 40 + \frac{(12-8) \times 20}{2 \times 12 - 8 - 11}$

$\qquad = 40 + \frac{4 \times 20}{24 - 19}$

$\qquad = 40 + \frac{80}{5}$

Mode = ₹ 56.

---

118. Find the unknown entries a, b, c, d, e and f in the following distribution of height of students in a class:

Height (in cm)	Frequency	Less than type c.f.
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
Total	50	

2013/2015   (3 Marks)

From the given table,

a = 12

b = 25 − 12 = 13

c = 25 + 10 = 35

d = 43 − c = 43 − 35 = 8

e = 48 − 43 = 5

f = 48 + 2 = 50.

---

119. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	X	20	15	y	5	60

2010/2012/2014/2016   (4 Marks)

Let us prepare the following table:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y$

Total $N = 45 + x + y = 60$

Here, N = 60 (given). So, $\frac{N}{2} = 30$.

Now, the median 28.5 belongs to the 20-30.

$\therefore$ Median class is 20-30

$\therefore$ $l = 20,\ f = 20,\ c.f. = 5 + x\ and\ h = 10$

Hence,   Median $= l + \left(\dfrac{\frac{N}{2} - cf}{f}\right) \times h$

$\Rightarrow$ $28.5 = 20 + \left(\dfrac{30 - 5 - x}{20}\right) \times 10$

$\Rightarrow 28.5 = 20 + \dfrac{25 - x}{2}$

$\Rightarrow 57 = 40 + 25 - x$

$\Rightarrow$ $57 = 65 - x$

$\Rightarrow x = 8$

Again, N = 60

$\Rightarrow 45 + x + y = 60$

$\Rightarrow 45 + 8 + y = 60$ $(\because x = 8)$

$\Rightarrow$ $y = 60 - 53 = 7$

Hence, $x = 8\ and\ y = 7$.

---

120. The frequency distribution of marks obtained by 36 students in a test is as follows:

Marks obtained	50-60	60-70	70-80	80-90	90-100
No. of students	4	8	12	6	6

Draw cumulative curves of 'less than type' and 'more than type' on the same axes and from them determine the median.

2015/2016   (4 Marks)

We first prepare 'less than type' and 'more than type' frequency distributions for the given data as follows:

Less than type:

Marks obtained	No. of students (frequency)
Less than 50	0
Less than 60	4
Less than 70	12
Less than 80	24
Less than 90	30
Less than 100	36

More than type:

Marks obtained	No. of students (Frequency)
More than and equal to 50	36
More than and equal to 60	32
More than and equal to 70	24
More than and equal to 80	12
More than and equal to 90	6
More than and equal to 100	0

Now, we draw the less than type ogive and more than type ogive corresponding to these two distributions as explained in earlier questions on the same graph paper as shown below:
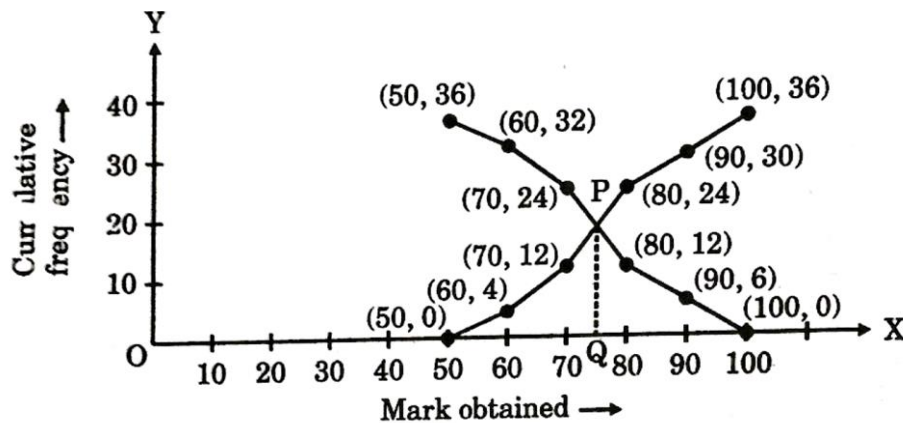
Note that the point of the two ogives is P.

Draw PQ ⊥ x-axes.

We observe that coordinates of Q are (75, 0)

So, median is = 75.

121. The given distribution shows number of wickets taken by the bowlers in one-day international cricket matches:

No. of wickets	No. of bowlers
Less than 15	2
Less than 30	5
Less than 45	9
Less than 60	17
Less than 75	39
Less than 90	54
Less than 105	70
Less than 120	80

Draw a 'less than type' ogive from the above data. Find median from the curve. Verify median by actual calculations.

2015/2016 (4 Marks)

Taking upper limits along the x-axis and cumulative frequencies (Number of bowlers) along y-axis, we draw the less than type ogive as follows:
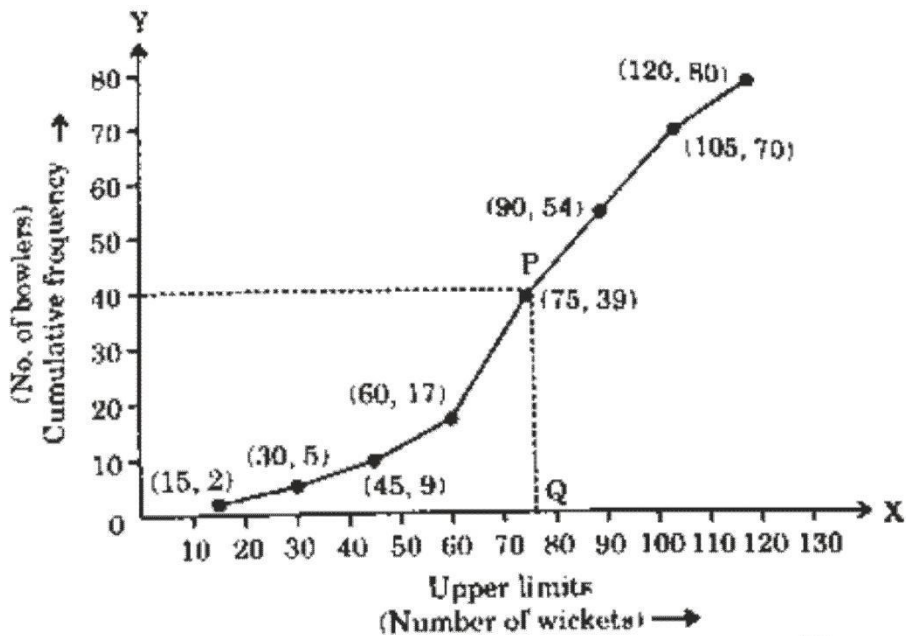
For finding the median, we draw a line through frequency $\frac{80}{2}$, i.e., 40, parallel to x-axis to intersect the ogive at P. then, we draw PQ $\perp$ x-axis. The x- coordinate of q is nearly 76. So, the median is 76 (approx.).

Verification: We convert the data into the following form:

Class	0-15	15-30	30-45	45-60	60-75	75-90	90-105	105-120
Frequencies	2	3	4	8	22	15	16	10

So, $\frac{N}{2} = \frac{80}{2} = 40$.

Hence, median class is 75-90.

So, median $= 75 + \frac{(40-39)}{15} \times 15$

$= 75 + 1 = 76$ (Verified)